FIXED EQUIPMENT NEWSLETTER

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APPENDIX 46

Starting with the 2019 edition, ASME Section VIII, Division 1 introduced earlier Code Case 2695 as Appendix 46 in the Code. This, combined with revision to paragraph UG-16(a), allowed the Division 1 pressure vessels to be designed using the provisions of Division 2 (with some restrictions). Appendix 46 often produces more economical Division 1 pressure vessel designs due to increased accuracy provided in Division 2 “Design by Rule” equations.

Appendix 46 also permits the use of “Design by Analysis” provided in Part 5 of Division 2 but only when the design rules are not available in Division 1 or Part 4 of Division 2. The restrictions that the Appendix imposes are that the allowable stresses shall be in accordance with UG-23, and that the weld joint efficiency shall be in accordance with UW-11 and UW-12. When using Part 4 “Design by Rule” equations for compressive loads, the allowable compressive stress is limited as prescribed in Division 2, 4.4.1.2 in lieu of UG-23(b).

There are several design rules provided in Part 4 of Division 2 that are not addressed in Division 1. Shallow cones, external loads on cylinders, cones and spheres are some of the examples. There is also a wide range of D_o/t values for compressive loads and external pressure, and external loads on flanges. Overall, the use of Appendix 46 provisions will lead to more favorable designs in many cases, even with the restrictions imposed.

In short, Appendix 46 provides an excellent opportunity for pressure vessel designers to use the more accurate design rules provided in Division 2 while designing Division 1 vessels. It is my firm belief that designers should evaluate the alternate designs provided in Division 2 and adopt them if it results in a more economical design.

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Featured in this issue…

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CoDesign Engineering provides a wide range of engineering services for heat exchangers, pressure vessels, storage tanks and piping design with a mission to achieve engineering excellence, customer satisfaction and innovative solutions. We bring over 50 years of combined experience with technical and commercial requirements of large capital projects and understanding of major design requirements across all engineering disciplines.

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- Review and certification of repair plans for pressure vessels
- Validation and certification of User Design Specifications and Manufacturer Design Reports
- Finite element analysis of pressure vessel components
- Registration of pressure vessels and accessories in Canadian provinces
- Design of lifting lugs and tailing lugs for tall columns
- Cyclic service analysis for pressure vessels per ASME Section VIII, Division 2, Part 5

Storage Tanks:

- Design, analysis and rerating of atmospheric storage tanks and low-pressure storage tanks based on API 650 and API 620 respectively

Training:

- Training: Process equipment and piping systems; Pressure vessels; Thermal and mechanical design of shell-and-tube heat exchangers; Aboveground storage tanks

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APPENDIX 47: REQUIREMENTS FOR PRESSURE VESSEL DESIGNERS

One of the major changes in the 2021 edition of ASME Section VIII, Division 1 (ASME VIII-1) is the minimum requirements for the individuals engaged in the design activities for pressure vessels. These requirements are mandated by the introduction of a new paragraph, U-2(b)(3), which states, “The Manufacturer has the responsibility of ensuring all personnel performing design activities are qualified in the applicable area(s) of design.” A new mandatory Appendix 47 has been added that describes the requirements for pressure vessel designers.

PERSON IN RESPONSIBLE CHARGE

The phrase “responsible charge” means personal accountability for ensuring that the design of pressure vessel certified by the Manufacturer meets the code, follows sound engineering design principles for those aspects not directly addressed by the code, and ensures that the vessel will operate safely in the intended service.

Appendix 47 requires the manufacturer to designate a designer, engineer or certifying engineer who will be in responsible charge of the design of the pressure vessel. This person may be a design service contractor engaged by the Manufacturer for their services. The person in responsible charge (PRC) shall be qualified to perform design work or exercise control of design work performed by others. The qualification and experience required of the PRC will depend on the design complexity of the pressure vessel.

The complexity of the pressure vessel includes factors such as design simplicity versus complexity , the types of materials and welding procedures used, the thickness of materials, the types of nondestructive examinations applied, and whether heat treatments are applied.

<u>CERTIFYING ENGINEER</u>

Certifying engineers shall be Chartered, Registered, of Licensed in accordance with one or more of the following:

a) As a Registered Professional Engineer in at least one state of the United States or at least one province of Canada.

b) With the International Register of Professional Engineers by an authorized member of the International Professional Engineers Agreement (IPEA).

c) With an authorized member of the Asia Pacific Economic Cooperation (APEC).

d) With an authorized member of the European Federation of National Engineering Associations (FEANI).

Certifying engineers shall have 4 years or more of experience in the design of pressure vessels.

<u>ENGINEER</u>

Engineer shall have a degree from an accredited university or college in engineering, science, or technology requiring an equivalent of 4 years of full-time study of higher education. In addition, engineer shall have 4 years or more of experience in the design of pressure vessels. *The Manufacturer may set a different requirement for the minimum number of years of experience in the design of pressure vessels.*

<u>DESIGNER</u>

Designer shall meet either of the following:

- The designer shall have completed an accredited engineering technician or associates degree, requiring the equivalent of at least 2 years of study, plus have a minimum of 6 years of experience in the design of pressure vessels.

- The designer shall have a minimum of 10 years of experience in the design of pressure vessels.

The Manufacturer may set a different requirement for the minimum number of years of experience in the design of pressure vessels.

As an alternative to requirements of Appendix 47, the Manufacturer may follow the requirements of ASME Section VIII, Division 2, Part 2.

The rules in Appendix 47 were intentionally written to accommodate the broad diversity of vessel manufacturing because there is no “one size fits all” solution for meeting these requirements. A simple pressure vessel manufacturer may only require a Designer designated as PRC for the limited scope of production they perform. He/she could be supervising any number of lesser qualified individuals performing basic design activities. Another simple pressure vessel manufacturer may not have any design staff members and may subcontract all the design activities to a design services contractor designated as PRC. Manufacturers having a broader scope of construction complexity and design may need a certifying engineer. The PRC shall be designated as such in the “Design” section of the QC system.

DESIGN SERVICES CONTRACTOR

Appendix 47 allows the PRC to be a design services contractor. It is not mandatory for the PRC to be an employee of the Manufacturer.

A letter of appointment shall be issued by the Manufacturer designating the design services contractor as the PRC to whatever level or extent the Manufacturer requires. This should be handles in the same manner as appointing an NDE contractor as a Level III examiner acting on behalf of the Manufacturer for NDE activities. This letter shall be signed and dated by the person signing the Statement of Authority and Responsibility in the QC System. The appointment letter shall also be countersigned and dated by the design services contractor acknowledging and accepting the appointment.

The design services contractor in PRC role could be the only person in responsible charge in some cases. In other cases, the contractor may be supplementing the existing designated design staff. In either case, he/she is not exempted from meeting the requirements of Appendix 47 for the designation and qualification.

QUALIFICATIONS FOR DESIGN ACTIVITY

The individuals performing design activities are not required to meet the qualifications for “Designer” or “Engineer” but shall work under the supervision of a PRC. The qualifications of individuals that perform the design activity for pressure vessels shall meet the minimum requirements set by the Manufacturer. These requirements shall be described in the Manufacturer’s Quality Control (QC) System and are as follows:

1) The individual shall have knowledge of the design requirements of ASME VIII-1 for the application of the Certification Mark with the appropriate designator.
2) The individual shall have knowledge of the Manufacturer’s quality program.
3) The individual shall have training commensurate with the scope, complexity, criticality, or special nature of the design activities to which oversight is to be provided.

The individual shall maintain a documented record containing objective evidence of meeting the qualifications for the experience and training obtained. He/she shall also be permitted to engage in any design activity required by ASME VIII-1 or any supplemental User’s Design Requirements (UDR).

The qualifications of the individual(s) performing the design activities shall be documented by the QC System, and if they are not qualified as “Designer” or “Engineer”, there shall be documented evidence of these individuals being permitted to engage in design activities only under supervision. Having another controlled document that designates individuals who have been qualified at this lower level for design activities under supervision is a good suggestion.

The following table lists design activities that require additional qualifications:

Design Activities	Code Location
Performance of numerical analysis [see ASME VIII-2, Part 2, 2.3.3.3(d)(2)(-f)]	Varies
Fatigue assessments a) Elastic stress analysis (see ASME VIII-2, Part 5, 5.1.2 and 5.5.3) b) Elastic-plastic stress analysis (see ASME VIII-2, Part 5, 5.5.4)	UG-22
Design due to seismic reactions a) Linear response history procedure b) Nonlinear response history procedure	UG-22
Quick-actuating closures	UG-35.2
Design not specifically addressed in ASME VIII-1	U-2(g)

The additional qualifications required for engaging in the design activities listed in the table above are as follows:

<u>CERTIFYING ENGINEER</u>

Certifying engineer may engage in or be in responsible charge of any of the design activities listed in the table.

<u>ENGINEER AND DESIGNERS</u>

Engineers and designers who engage in or be in responsible charge of any of the design activities listed in the table shall have evidence of additional qualifications as follows:

Numerical Analysis:

a) 2 years or more of experience performing design analysis.

b) Have received instruction in the use and understanding of any numerical analysis computer programs from one of the following:

1. The software vendor
2. A training course acceptable to the software developer
3. A certifying engineer with requisite knowledge of the computer program and qualifications to train others on its use

Fatigue Assessments:

a) 2 years or more of experience performing fatigue assessments.

b) The individual shall be working under the responsible charge of a certifying engineer.

Other Design Activities:

a) 2 years or more of experience performing seismic reactions, designing quick-actuating closures, or being engaged in U-2(g) design activities.

Fatigue analysis is the only activity that absolutely requires a Certifying Engineer to be involved, either performing the analysis of supervising their performance by others as the PRC. For all other design activities, a Certifying Engineer is not required if the qualification requirements for an Engineer or a Designer along with any additional qualification requirements (when applicable) are met, and the individual is designated as PRC in the Manufacturer's QC System.

MANUFACTURER'S RESPONSIBILITIES

The manufacturer shall:

1) Identify the minimum qualifications required for design of their products
2) Ensure that the required qualifications have been met
3) Set requirements for the frequency of activity engagement to maintain the required qualifications, and
4) Describe the requirements in the QC System.

While establishing requirements for the PRC for design activities, the manufacturer should consider the following body of knowledge elements for this person:

<u>BASIC CAPABILITY</u>

a) Mathematics

<u>TECHNICAL CAPABILITY</u>

a) Manufacturing
b) Design
c) Engineering science
d) Engineering tools
e) Quality control and quality assurance
f) Technical breadth
g) Technical depth

<u>PROFESSIONAL PRACTICE</u>

a) Communication
b) Legal aspects of engineering
c) Continuing education

The manufacturer is responsible for designating the PRC. The PRC is assigned the authority and responsibility for their activities in the "Design" section of the QC System, usually by the title, such as "Designer", "Engineer", or "Certifying Engineer" as appropriate.

References:

ASME Boiler and Pressure Vessel Code, Section VIII, Division 1: Edition 2021

DETERMINATION OF ASSEMBLY BOLT STRESS

The procedures for determination of an appropriate assembly bolt stress that maintains the required joint integrity of a flange is provided in ASME PCC-1 Appendix O (Appendix O). These procedures require controlled assembly methods. Appendix O provides both a simple approach (may result in damage to joint components) and a joint component approach (that considers the integrity of all joint components).

ASSUMPTIONS

1) Joint component condition (flange surface finish, bolt spacing, flange rigidity, bolt condition, etc.) are within acceptable limits.

2) Gasket being used undergoes a reasonable amount (>15%) of relaxation during the initial stages of operation.

   The effects of operational loads in increasing the bolt stress need not be considered (i.e., gasket relaxation will exceed any operational bolt load increase. In some rare cases, this may not be the case, and the limits should then also be checked at both the ambient and operating bolt stress and temperatures.

3) Material is ductile – strain at tensile failure is more than 15%.

   For brittle materials, the margin between the specified assembly bolt stress and the point of component failure may be considerably reduced and, therefore, additional safety factors should be introduced to guard against such failure.

4) The method does not consider the effects of fatigue, creep, or environmental damage mechanisms on either the bolt or the flange

   These additional modes of failure may also need to be considered for applications where they are found and additional reductions in assembly bolt stress may be required to avoid joint component failure.

ASSEMBLY BOLT STRESS SELECTION

The bolt stresses must be established with due consideration to the following joint integrity issues:

a) <u>Sufficient Gasket Stress to Seal the Joint</u>

   The assembly bolt stress should provide sufficient gasket stress to seat the gasket and sufficient gasket stress during operation to maintain a seal.

b) <u>Damage to Gasket</u>

   The assembly bolt stress should not be so high as to cause over-compression (physical damage) of the gasket or excessive flange rotation of the flange, which can also lead to localized gasket over-compression.

c) <u>Damage to the Bolts</u>

   The specified bolt stress should be below the bolt yield point such that bolt failure does not occur. In addition, the life of bolt can be extended by specifying an even lower load.

d) <u>Damage to the Flange</u>

   The assembly bolt stress should be selected such that permanent deformation of flange does not occur. If the flange is deformed during assembly, then it is likely that it will leak during operation or that successive assemblies will not be able to seal due to excessive flange rotation. Leakage due to flange may be due to the concentration of the gasket stress on the gasket outside diameter causing damage or additional relaxation. Another potential issue is the flange face outer diameter touching, which reduced the effective gasket stress.

However, it is also important to consider the practicalities involved with the in-field application of the specified bolt stress. If a different assembly stress is specified for each flange in a plant, including all variations of standard piping flanges, then it is unlikely, without a significant assembly quality assurance plan, that success will actually be improved in the field by comparison to a simpler method. Depending on the complexity of the joints in a given plant, a simple approach (standard bolt stress per size across all standard flanges, for example) may actually be more effective in preventing leakage than a more complex approach that includes consideration of the integrity of all joint components.

SIMPLER APPROACH

The appropriate bolt stress for a range of typical joint configurations may be determined as:

$Sb_{sel} = Sg_T \frac{A_g}{n_b A_b}$ Where, Sb_{sel} = Selected assembly bolt stress, psi (MPa)

Sg_T = Target assembly gasket stress, psi (MPa)

A_g = Gasket area, in^2 (mm^2)

n_b = Number of bolts

A_b = Bolt root area, in^2 (mm^2)

Note: The gasket area is calculated as $\pi/4(G_{OD}^2 - G_{ID}^2)$ where G_{OD} is the gasket outside diameter and G_{ID} is the gasket inside diameter. When the gasket has additional gasket area such as pass partition gasket, which may not be as compressed as the main outer sealing element due to flange rotation, then a reduced portion of that area, such as half the additional area, should be added to A_g.

The average bolt stress across joints considered may then be selected and this value can be converted into a torque table using the following equation:

$T_b = Sb_{sel} K A_b \phi_b / 12$ for US Customary units, and

$T_b = Sb_{sel} K A_b \phi_b / 100$ for metric units.

Where, T_b = Assembly bolt torque, ft-lb (N.m)

K = Nut factor (for bolt material and temperature)

Φ_b = Bolt diameter, in (mm)

An example of the type of table produced using this method is given in Table 1 at the end of this article, which was constructed using a bolt stress of approximately 50,000 psi and a nut factor, K, of approximately 0.20 with adjustments made based on industry experience.

JOINT COMPONENT APPROACH

REQUIRED INFORMATION

There are several values that must be known prior to calculating the appropriate assembly bolt stress using this approach.

a) The maximum permissible flange rotation (θg_{max}) at the assembly gasket stress and the gasket operating temperature must be obtained from the industry test data or from the gasket manufacturer. There is presently no standard test for determining this value; however, typical limits vary from 0.3 deg for expanded

PTFE gaskets to 1.0 deg for typical graphite filled metallic gaskets (per flange). A suitable limit may be determined for a given site based on calculation of the amount of rotation that presently exists in flanges in a given service using the gasket type in question.

b) The maximum permissible bolt stress (Sb_{max}) must be selected by the user. This value is intended to eliminate damage to the bolt or assembly equipment and may vary from site to site. It is typically in the range of 40% to 70% of the ambient bolt yield stress.

c) The minimum permissible bolt stress (Sb_{min}) must be selected by the user. This value is intended to provide a lower limit such that bolting inaccuracies do not become a significant portion of the specified assembly bolt stress, Sb_{sel}. This value is typically in the range of 20% to 40% of the ambient bolt yield stress.

d) The maximum permissible bolt stress for the flange (Sf_{max}) must be determined, based on the particular flange configuration. Additionally, when the limits are being calculated, the flange rotation at that load should also be determined (θf_{max}).

e) The target assembly gasket stress (Sg_T) should be selected by the user in consultation with the gasket manufacturer. The target gasket stress should be selected to be towards the upper end of the acceptable gasket stress range, as this will give the most amount of buffer against joint leakage.

f) The maximum assembly gasket stress (Sg_{max}) must be obtained from industry test data or from the gasket manufacturer. This value is the maximum compressive stress at the assembly temperature, based on full gasket area, which the gasket can withstand without permanent damage (excessive leakage or lack of elastic recovery) to the gasket sealing element. Any value provided should include consideration of the effects of flange rotation for the type of flange being considered in increasing the gasket stress locally on the outer diameter.

g) The minimum gasket seating stress (Sg_{min-S}) must be obtained from industry test data or from the gasket manufacturer. This value is the minimum recommended compressive stress at the assembly temperature and is based on full gasket area. The value is the stress that the gasket should be assembled to in order to obtain adequate redistribution of any filler materials and ensure an initial seal between the gasket and the flange faces.

h) The minimum gasket operating stress (Sg_{min-O}) must be obtained from industry test data or from the gasket manufacturer. This value is the minimum recommended compressive stress during operation and is based on full gasket area. This is the gasket stress that should be maintained on the gasket during operation in order to ensure the leakage does not occur.

i) The gasket relaxation fraction (ϕ_g) must also be obtained from industry test data or from the gasket manufacturer for the gasket in flange assemblies of similar configuration to the ones being assessed. A default value of 0.7 may be used if data are not available.

<u>DETERMINING THE APPROPRIATE BOLT STRESS</u>

Once the limits are defined, it is possible to utilize the following process for each joint configuration. This process can be performed using a spreadsheet or software program, which allows the determination of many values simultaneously.

Step 1:

Determine the target bolt stress.

$$Sb_{sel} = Sg_T \frac{A_g}{n_b A_b}$$

Step 2:

Determine if the bolt upper limit controls.

$Sb_{sel} = \min.(Sb_{sel}, Sb_{max})$

Step 3:

Determine if the bolt lower limit controls.

$Sb_{sel} = \max.(Sb_{sel}, Sb_{min})$

Step 4:

Determine if the flange limit controls.

$Sb_{sel} = \min.(Sb_{sel}, Sf_{max})$

Step 5:

Check if gasket assembly seating stress is achieved.

$Sb_{sel} \geq Sg_{min\text{-}S}[A_g/(A_b n_b)]$

Step 6:

Check if gasket operating stress is maintained.

$Sb_{sel} \leq (Sg_{min\text{-}O}\, A_g + \pi/4 P_{max}\, G_{ID}^2) / (\phi_g A_b n_b)]$

Step 7:

Check if gasket maximum stress is exceeded.

$Sb_{sel} \leq Sg_{max}\, [A_g/(A_b n_b)]$

Step 8:

Check if flange rotation limit is exceeded.

$Sb_{sel} \leq Sf_{max}\, (\theta g_{max}/\theta f_{max})$

If one of the final checks (steps 5 through 8) is exceeded, then judgment should be used to determine which controlling limit is more critical to integrity and, therefore, what the selected load ought to be.

References:

1) ASME PCC-1: Guidelines for Pressure Boundary Bolted Flange Joint Assembly

HEADS USED IN PRESSURE VESSELS

Semi-Ellipsoidal Head, Hemispherical Head and Torispherical Head are three types of formed ASME Pressure Vessel Dished Heads, as shown in Figure 1 below. Additionally, flat head may also be used in the construction of pressure vessels.

Semi-Ellipsoidal head

This is also called a 2:1 elliptical head. The shape of this head is more economical because the height of the head is just a quarter of the diameter. Its radius varies between the major and minor axis.

Hemispherical head

A sphere is the ideal shape for a head because the pressure in the vessel is divided equally across the surface of the head. The radius (R) of the head equals the radius of the cylindrical part of the vessel.

Torispherical head

These heads have a dish with a fixed radius (CR), the size of which depends on the type of torispherical head. The transition between the cylinder and the dish is called the knuckle. The knuckle has a toroidal shape.

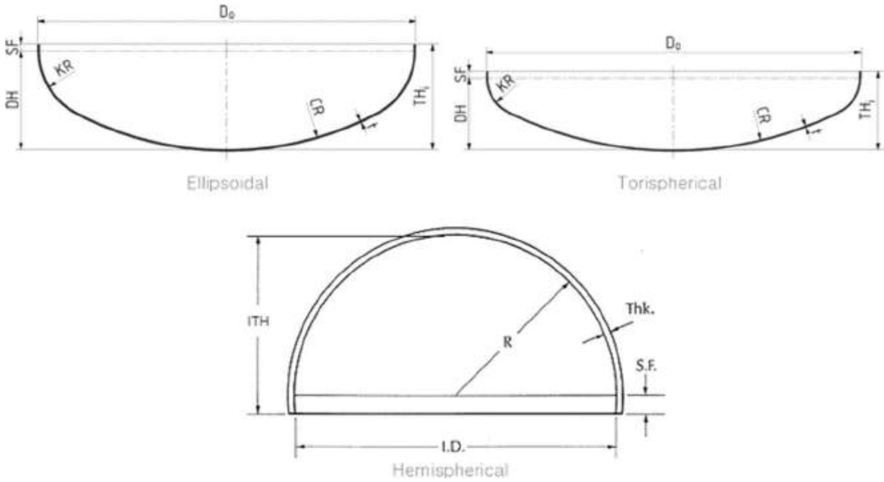


Figure 1: Pressure Vessel Heads

SEMI-ELLIPSOIDAL HEAD

This elliptical shape head is designed in a 2:1 ratio, and it is also known as 2:1 head. The semi-ellipsoidal head features a half ellipse, so its head depth is usually quarter of its diameter. Typically, pressure vessel fabricators design these heads with three radii, which is approximated to an ellipse. The ellipse usually has a smallest outer diameter, a large crown, and an intermediate radius. The code recommends the approximation of semi-ellipsoidal head using two radii (which would normally be considered an ASME F&D head), one with crown radius equal to 0.90D and with knuckle radius equal to 0.17D.

This head shape is economical and is perfect for high pressures owing to its height to weight ratio. Semi-elliptical head has radius varying between the minor and major axis, in the ratio of 2:1. This varying ratio allows it to hold gases or fluids at high pressure. These heads are made of thinner materials such as a flat plate, which help reduce their fabrication costs.

The ASME Code formula for semi-ellipsoidal heads are such that the required thickness is same as that required for a cylinder with same inside diameter.

HEMISPHERICAL HEAD

These heads have a simple radial geometry, where the head depth is half the diameter. Storage tanks made from hemispherical heads feature two pressure heads that are placed back-to-back, thereby creating a storage sphere, which efficiently stores materials at high pressure. Hemispherical heads feature thinner heads than shell and use a standard code 3:1 that taper at the transition. The transition is part of the stronger head, the shell is not tapered down on the straight section because it needs the full thickness. For same design pressures, design temperature and material, the calculated wall thickness under internal pressure for a hemispherical head will be approximately half of the shell thickness.

Normally, hemispherical heads are made from welded pieces rather than from flat sheets used in other head types mentioned here. This makes it the thinnest head, but sometimes the most expensive one. These heads are recommended for applications that demand high-pressure storage or large diameter vessels.

TORISPHERICAL HEAD

Also known as ASME F&D heads, these heads are ideal for pressure vessels designed for moderate pressures. They have a crown radius equal to the outside diameter of the head ($r_1 = D_o$), and a knuckle radius equal to 6% of the outside diameter ($r_2 = 0.06xD_o$). ASME Section VIII, Division 1 does not allow the knuckle radius to be less than 6% of the outside diameter, nor does it allow the crown radius to be more than the outside diameter of the straight flange portion of the head. ASME F&D heads possess a flatter or lower profile, which makes them ideal for pressure vessels that have a height restriction. They are shallower than 2:1 ellipsoidal head, which means they need not be dished as much and as deep as a 2:1 head.

Based on design pressure and vessel size, ASME F&D heads are thicker than 2:1 elliptical heads. This is best explained through the example below:

- ASME F&D 60″ head would need a 69″ circle blank to form the head. You need to add 14% to OD of the requested head for a blank size.
- ASME 2:1 60″ elliptical head would need a 72” circle blank to form the head. You need to add 20% to OD of the requested head for blank size.

In the forming phase, the material of the head flows to the straight flange or outer flange, thereby making the heads thicker. The head possesses an offset in the head-to-shell circumferential seam, which is usually managed at 3:1, as per the Code.

For same design pressures, design temperature and material, the calculated wall thickness under internal pressure for an ASME F&D head will be approximately equal to 1.77 times the shell thickness.

FLAT HEAD

Pressure vessels may also be provided with flat heads. These heads have flat surfaces, which makes them ideal for applications that demand flat inside surfaces. Flat heads are considered ideal for no pressure applications; when used for pressure applications, these pressure vessel heads may be quite expensive than other heads discussed here. This is why flat heads are mainly used for holding tanks or storage tanks that store materials at no pressure. These heads are also recommended in applications where lower head heights than ASME F&D heads are needed.

Sometimes, thin flat plate supported by exterior beams across the width of the pressure vessel is provided to reap the benefits of using a flat head without the huge weight of steel a flat head requires.

COMPARISON BETWEEN SEMI-ELLIPSOIDAL, HEMISPHERICAL, ASME F&D, AND FLAT HEADS

A cylindrical shell made of 0.500" thick SA-516-70 material (rated to 20,000 psi at 100°F) is rolled to 48" OD. The inside diameter (ID) ends up at 47". This cylinder and the seams joining it to any attached heads are fully radiographed, and there is no corrosion allowance. The ASME VIII-1 calculated design pressure for the cylinder is 420 psi.

For this comparison, each of the four heads is attached to the cylinder, with diameters matching on the ID. The wall thickness is varied to meet the 420 psi rating of the cylinder. The results – thickness, height, volume, and weight of one head only – are shown in the tabulation below and in Figure 2 (semi-ellipsoidal and ASME F&D heads include 1-1/2" straight flange):

Head	Thickness (in)	Outside Height (in)	Volume (US Gallon)	Weight (lbs)
Cylinder	0.5	---	---	---
Hemispherical	0.2474	23.75	117.7	245.5
Semi-Ellipsoidal	0.4947	13.74	70.1	397.3
ASME F&D	0.8901	10.29	47.7	602.9
Flat	3.9120	3.91	0	1920.8

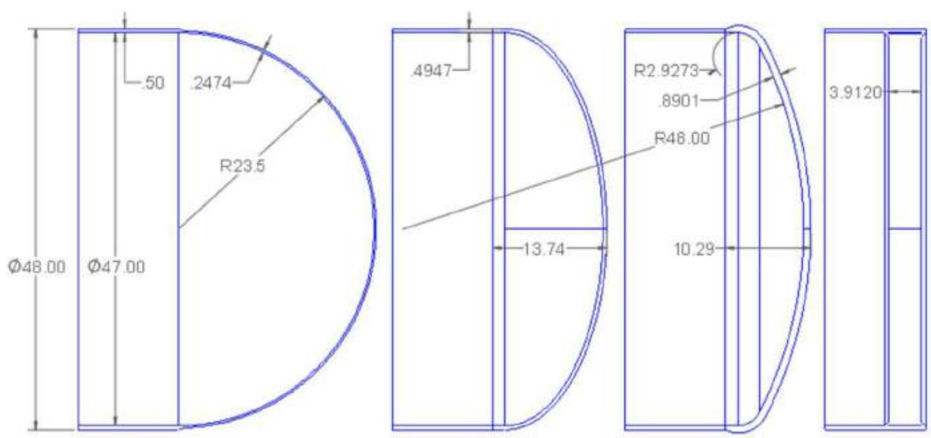


Figure 2: Comparison of Four Types of Heads
(*Courtesy Pressure Vessel Engineering*)

References:

Introduction to Spherical and Cylindrical Pressure Vessels – *Explore the World of Piping*

Comparison Between Head Types: Hemi. SE, F&D and Flat – *Pressure Vessel Engineering*

CoDesign Engineering LLC (Houston, TX) and Universal Technical Systems (Loves Park, IL) have entered into an agreement to develop software products for pressure vessels, heat exchangers and storage tanks. Design software and online learning modules for various components will be available in coming weeks and months. All products are designed to be conveniently accessible from desktop browser, smartphone, or a tablet.

Stay tuned for the announcement of the first product for the "Design of Bolted, Integral Weldneck Flanges" in the February issue of the newsletter.

CONE-TO-SHELL JUNCTION ANALYSIS

This article is brought to you courtesy of Ray Delaforce. Ray is an engineer at Hexagon/ Intergraph and lives in Houston, Texas.

When considering the cone to shell junction, if care is not taken in getting the components correctly and adequately designed, collapse of the junction can occur. This applies whether the junction is subject to internal pressure and/or external pressure. In the case of internal pressure, the mode of collapse is not intuitive. It is very easy to imagine the internal pressure could destabilize the junction. However, this is not the case.

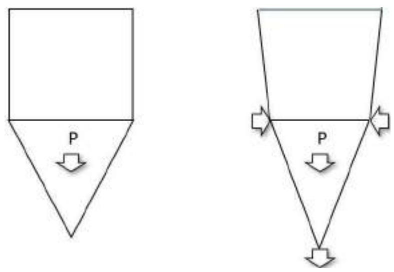


Figure 1: Pressure Forces Acting on a Cone

In Figure 1, the downward component of pressure pushes the cone downward causing the either side of the cone to be pulled inward. That causes a compressive hoop stress in the region of the junction tending to cause junction collapse. We can see the forces acting at the junction in Figure 2, from the free body diagram, giving rise to the problem.

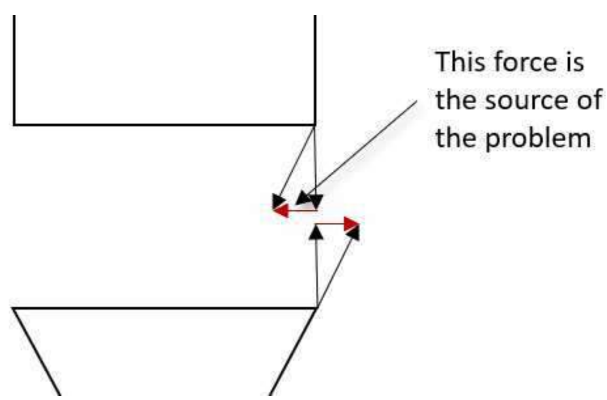


Figure 2: Cone-to-Shell Junction – Free Body Diagram

There are several ways the integrity of the junction can be assured:

1. Use thicker components at the junction, or,
2. Provide a reinforcing ring as detailed in Figure 3

Figure 3 shows the vessel, specifically, the cone-to-shell junctions we are going to analyze. There is a junction at the large end of the cone, and one at the small end of the cone. Note the reinforcing ring in the region of the large diameter of the cone. This is the data for vessel we are going to analyze:

All dimensions in mm				Material: SA-516 70
				Temperature: 100°C
	P	=	1 MPa	Internal pressure
	P_e	=	0.1 MPa	External pressure
	E_1	=	1.0 all	Joint factors
	E_S	=	198 163 MPa	Elastic modulus shells
	E_r	=	198 163 MPa	Elastic modulus of the ring
	S	=	138 MPa	Allowable stress of all components

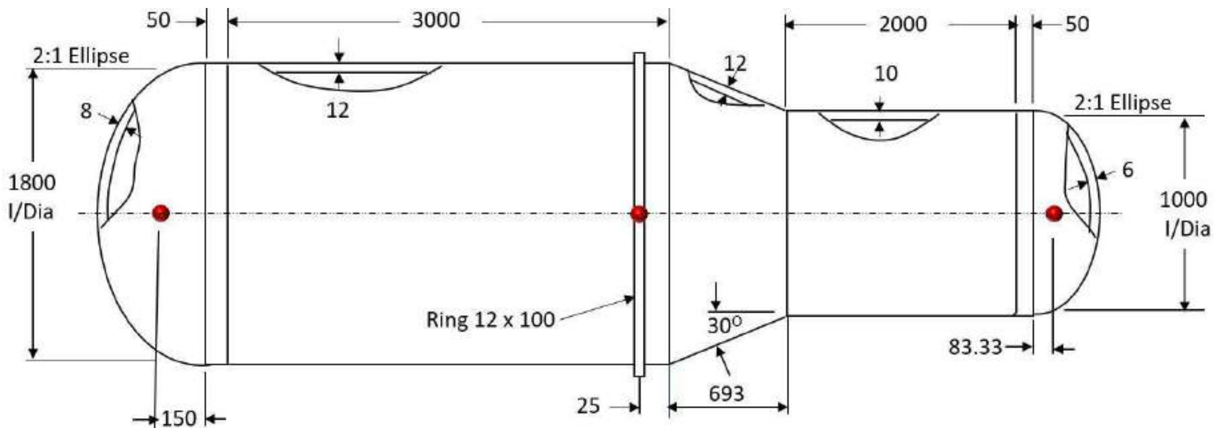


Figure 3: Vessel with a Cone-to-Shell Junction

The three dots represent points of support, which are important when we come to analyze the junctions for external pressure. They are of no consequence when considering the analysis for internal pressure.

ANALYSIS FOR INTERNAL PRESSURE per Appendix 1-5

Analysis of the large end of the cone

$$\alpha = \text{atan}\left(\frac{D_{Li} - D_{Si}}{2 \cdot L}\right) = \text{atan}\left(\frac{1800 - 1000}{2 \cdot 693}\right) = 30^o \;\; \text{note, inside diameters}$$

This represents the slope length of the cone as shown in the code, equation in the nomenclature:

$$L_c = \sqrt{L^2 + (R_L - R_S)^2} = 800\ mm$$

The value is Δ equation (1):

$$\Delta = 326.6 \cdot \sqrt{\frac{P}{S \cdot E_1}} \quad = 326.6 \cdot \sqrt{\frac{1.0}{138 \cdot 1.0}} = 27.802^o$$

The value of y – equation in the nomenclature:

$$y = S \cdot E_s = 138 \cdot 198\,163 = 27\,346\,494\ MPa^2$$

The value is k – equation in the nomenclature

$$k = \frac{y}{S \cdot E_r} = 1 \text{ this is fairly obvious}$$

Compute the required thickness of the large shell and the large end of the cone for internal pressure:

$$t_s = \frac{P \cdot R_{iL}}{S \cdot E_1 = 0.6 \cdot P} = \frac{1.0 \cdot 900}{138 \cdot 1.0 - 0.6 \cdot 1.0} = 6.55\ mm$$

$$t_r = \frac{P \cdot R_{iL}}{S \cdot E_1 = 0.6 \cdot P} \cdot \frac{1}{cos(\alpha)} = \frac{1.0 \cdot 900}{138 \cdot 1.0 = 0.6 \cdot 1.0} \cdot \frac{1}{cos(30^o)} = 7.5631\ mm$$

Compute the end load on the cone and the required area of the cone junction – equation (2):

$$Q_L = \frac{P \cdot R_{iL}}{2} = \frac{1.0 \cdot 900}{2} = 450\ {}^{N}/_{mm}$$

$$A_{rL} = \frac{k \cdot Q_L \cdot R_{iL}}{S \cdot E_1} \cdot \left(1 - \frac{\Delta}{\alpha}\right) tan(\alpha) = \frac{1.0 \cdot 450 \cdot 900}{138 \cdot 1.0} \cdot \left(1 - \frac{27.802}{30}\right) tan(30^o) = 123.772\ mm^2$$

Compute the actual junction area – equation (3):

$$A_{eL} = (t_L - t_s) \cdot \sqrt{R_{iL} \cdot t_L} + (t_c - t_r) \cdot \sqrt{\frac{R_L \cdot t_c}{cos(\alpha)}}$$

$$= (12 - 6.55) \cdot \sqrt{900 \cdot 12} + (12 - 7.5725) \cdot \sqrt{\frac{900 \cdot 12}{cos(30^o)}} = 1061\ mm^2$$

Compute the required junction area – equation (10):

$$A_{rL} = \frac{k \cdot Q_L \cdot R_L}{S_s \cdot E_1} \cdot \left(1 - \frac{\Delta}{\alpha}\right) \cdot tan(\alpha) = \frac{1.0 \cdot 450 \cdot 900}{138 \cdot 1.0} \cdot \left(1 - \frac{\Delta}{30}\right) \cdot tan(30^o)$$

$$= 123.77\ mm^2$$

The junction area is satisfactory

Decay length of the shell and cone:

$$l_s = 0.55\sqrt{D_i \cdot t_L} = 0.55\sqrt{1800 \cdot 12} = 80.833\ mm$$

$$l_c = 0.55\sqrt{D_i \cdot \frac{t_c}{cos(\alpha)}} = 0.55\sqrt{1800 \cdot \frac{12}{cos(30^o)}} = 86.858\ mm$$

The junction area is adequate.

<u>Analysis of the small end of the cone for internal pressure – equation 6:</u>

$$\Delta = 89\sqrt{\frac{P}{S_s \cdot E_1}} = 89\sqrt{\frac{1.0}{138 \cdot 1.0}} = 7.5762$$

Compute the required thickness for the small end junction for internal pressure:

$$t_s = \frac{P \cdot R_i}{S \cdot E_1 = 0.6 \cdot P} = \frac{1.0 \cdot 500}{138 \cdot 1.0 - 0.6 \cdot 1.0} \qquad = 3.639\ mm$$

$$t_r = \frac{P \cdot R_i}{S \cdot E_1 = 0.6 \cdot P} \cdot \frac{1}{cos(\alpha)} = \frac{1.0 \cdot 500}{138 \cdot 1.0 = 0.6 \cdot 1.0} \cdot \frac{1}{cos(30^o)} = 4.2017\ mm$$

Compute Q_S – equation in the nomenclature:

$$Q_S = \frac{P \cdot R_s}{2} = \frac{1.0 \cdot 500}{2} = 250\ {}^N/_{mm}$$

Compute the actual quired junction area – equation (8):

$$A_{es} = 0.78 \cdot \left(\sqrt{R_S \cdot t_s} \cdot (t_s - t) + \sqrt{R_s \cdot \frac{t_c}{cos(\alpha)}} \cdot (t_c - t_r) \right)$$

$$= 0.78 \cdot \left(\sqrt{500 \cdot 10} \cdot (10 - 3.639) + \sqrt{500 \cdot \frac{12}{cos(30^0)}} \cdot (12 - 4.2017) \right)$$

$$= 857.117\ mm^2$$

Compute the required junction area – equation (7):

$$A_{rS} = \frac{k \cdot Q_S \cdot R_S}{S \cdot E_1} \left(1 - \frac{\Delta}{\alpha}\right) tan(\alpha) = \frac{1.0 \cdot 250 \cdot 500}{138 \cdot 1.0} \left(1 - \frac{7.5762}{30}\right) tan(30^o)$$

$$= 390.764\ mm^2$$

The area is adequate.

ANALYSIS FOR EXTERNAL PRESSURE per Appendix 1-8

Analysis is the large end of the cone:

As we are now working with the outside dimension:

D_L	=	1824 mm	Outside diameter of the large shell
D_S	=	1020 mm	Outside diameter of the small shell
R_L	=	912 mm	Outside radius of the large shell
R_S	=	510 mm	Outside radius of the small shell

Calculation starts here:

$$\Delta = 104 \sqrt{\frac{P_e}{S_s \cdot E_1}} = 104 \sqrt{\frac{0.1}{138 \cdot 1.0}} = 2.8$$

Now, the minimum thickness for the external pressure as computed by PV Elite are given here. It would take too long to derive them in this document. These are the minimum computed thickness to sustain the external pressure:

t = 7.284 mm for the large shell

t_S = 4.381 mm for the small shell

t_r = 3.480 mm for the cone

According to UG-99, the thickness used in the external was multiplied by cos(α). So, to restore the thickness we must make this adjustment:

$$t_r = \frac{t_r}{cos(\alpha)} = \frac{3.480}{cos(30^o)} = 4.0181\ mm$$

<u>Compute L_L, the effective distance of the shell between the points of the support:</u>

$$L_L = 3000 - 25 + 50 + 150 = 3175\ mm \quad \text{see Figure 1}$$

<u>Compute A_{TL} and Q_L, equations in the nomenclature:</u>

$$A_{TS} = \frac{L_L \cdot t_L}{2} + \frac{L_c \cdot t_c}{2} = \frac{3175 \cdot 12}{2} + \frac{800 \cdot 12}{2} = 25050\ mm^2$$

$$Q_L = \frac{P_e \cdot R_L}{2} = \frac{0.1 \cdot 912}{2} = 45.6\ {}^{N}\!/_{mm} \quad \text{there is now force } f$$

<u>Compute the required area in the junction:</u>

$$A_{rL} = \frac{k \cdot Q_L \cdot R_L \cdot tan(\alpha)}{S_s \cdot E_1}\left(1 - \frac{1}{4}\left(\frac{P_e \cdot R_L - Q_L}{Q_L}\right) \cdot \frac{\Delta}{\alpha}\right)$$

$$= \frac{1.0 \cdot 45.6 \cdot 912 \cdot tan(30^o)}{138 \cdot 1.0}\left(1 - \frac{1}{4}\left(\frac{0.1 \cdot 912 - 45.6}{45.6}\right) \cdot \frac{2.8}{30}\right)$$

$$= 169.884\ mm^2$$

<u>Compute the actual area available in the junction:</u>

$$A_{eL} = 0.55\left(\sqrt{D_L \cdot t_L} \cdot (t_L - t) + \sqrt{D_L \cdot \frac{t_c}{cos(\alpha)}} \cdot (t_c - t_r)\right)$$

$$= 0.55\left(\sqrt{912 \cdot 12} \cdot (12 - 7.284) + \sqrt{912 \cdot \frac{12}{cos(30^o)}} \cdot (12 - 4.0181)\right)$$

$$= 1082\ mm^2$$

<u>Computed required area in the junction:</u>

$$A_{rL} = \frac{k \cdot Q_L \cdot R_L \cdot tan(\alpha)}{S_s \cdot E_1} \cdot \left(1 - \frac{1}{4} \cdot \left(\frac{P_e \cdot R_L - Q_L}{Q_L}\right) \cdot \frac{\Delta}{\alpha}\right)$$

$$= \frac{1.0 \cdot 45.6 \cdot 912 \cdot tan(30^o)}{138 \cdot 1.0} \cdot \left(1 - \frac{1}{4} \cdot \left(\frac{0.1 \cdot 912 - 45.6}{45.6}\right) \cdot \frac{2.8}{30}\right)$$

$$= 169.88\ mm^2$$

The area is adequate.

Now we have to consider the moments inertia[1] for the cone junction. We need some intermediate values to compute the required moment of inertia as we must interrogate Figure CS-2 from ASME Section II – Part D.

$$M = \frac{-R_L \cdot tan(\alpha)}{2} + \frac{L_L}{2} + \frac{{R_L}^2 - {R_S}^2}{3 \cdot R_L \cdot tan(\alpha)}$$

$$= \frac{-912 \cdot tan(30^0)}{2} + \frac{3178}{2} + \frac{912^2 - 510^2}{3 \cdot 912 \cdot tan(30^o)}$$

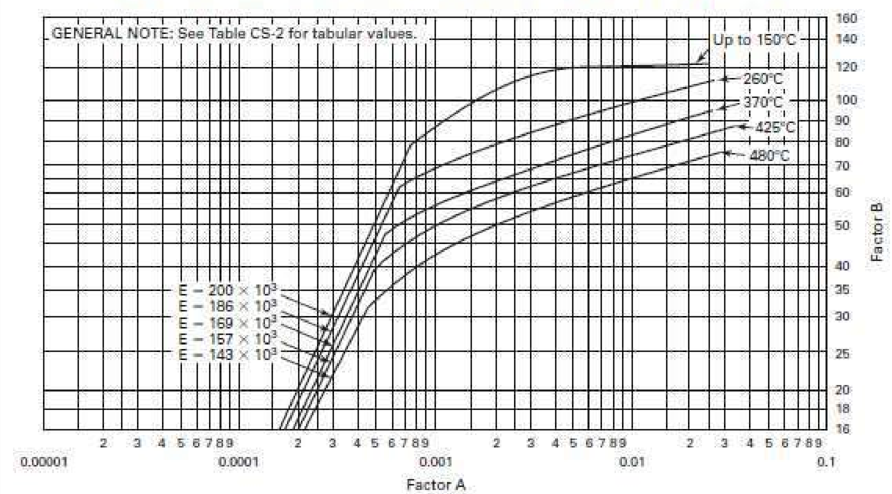
$$= 1686.275\ mm$$

$$F_L = P_e \cdot M = 0.1 \cdot 1686.275 = 168.628\ {}^{N}/_{mm}$$

$$B = \frac{3}{4} \cdot \frac{F_L \cdot D_L}{A_{TL}} = \frac{3}{4} \cdot \frac{168.628 \cdot 912}{25\ 050} = 9.2085\ MPa$$

With that value of B, we can now obtain the strain value of A from Figure CS-2:

Figure CS-2
Chart for Determining Shell Thickness of Components Under External Pressure Developed for Carbon or Low Alloy Steels With Specified Minimum Yield Strength 207 MPa and Higher



GENERAL NOTES:
(a) The external pressure charts do not account for reduction of buckling strength due to creep under long-term loads. The effect of creep on buckling shall be considered at temperatures for which allowable stresses are shown italicized in Tables 1A, 1B, 5A, and 5B, and in boldface in Tables 2A and 2B.
(b) The external pressure chart assigned for a particular material is obtained from the stress tables 1A, 1B, 2A, 2B, 5A, and 5B under the column for External Pressure Chart No. for that material and is mandatory, with the exception of Tables 5A and 5B.

As can be seen from the above chart, A cannot be read directly as the value of B is lower that the lowest value available which is 16 MPa. A must be computed another way. The method shown in Appendix 1-8 is as follows:

$$A = \frac{2 \cdot B}{E_s} = \frac{2 \cdot 9.2085}{198\ 163} = 0.000\ 929\ {}^{mm}/_{mm}$$

Now we can compute the required moment of inertia at the cone junction:

[1] The moment of inertial is probably better known as the second moment of area

$$I_S' = \frac{A \cdot D_L{}^2 \cdot A_{TL}}{10.9} = \frac{0.000\ 092 \cdot 1824^2 \cdot 25\ 050}{10.9}$$

$$= 710\ 632\ mm^4$$

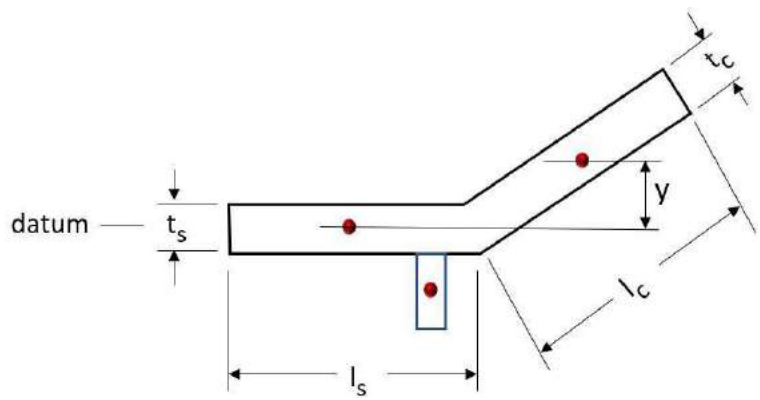
Now we have the tricky task of analysing and actual moment of inertia of the junction. In the Appendix I have done the derivation for a slanting component (the cone) to derive its section properties. The goal is to determine the actual moment of inertia. You can follow along as we develop the procedure.

First, obtain the decay lengths of the shell and cone:

$$l_S = 0.55\sqrt{D_L \cdot t_L} = 0.55\sqrt{912 \cdot 12} \qquad = 81.370\ mm$$

$$l_c = 0.55\sqrt{D_L \cdot \frac{t_c}{cos(\alpha)}} = 0.55\sqrt{912 \cdot \frac{12}{cos(30^o)}} = 87.435\ mm$$

To help you visualise the setup for computing the actual moment of inertia of the junction, this illustration should help:



The datum is the line from which all the dimensions to the component centroids is taken. The illustration shows the shell, cone and ring. Looking at the Appendix should enable you to see the procedure. Breaking down the components in the above illustration. We first compute the area of the components:

$$A_{shell} = l_s \cdot t_s \quad = 81.3703 \cdot 12 \ = 976.4436 \ \ mm^2 \qquad \text{area of the shell}$$

$$A_{cone} = l_c \cdot t_c \quad = 87.435 \cdot 12 \ = 1049.222 \ \ mm^2 \qquad \text{area of the cone}$$

$$A_{ring} = h \cdot t_{ring} = 100 \cdot 12 \qquad = 1200 \qquad mm^2 \qquad \text{area of the ring}$$

We need the distance from the datum to the centroid of the cone:

$$y = \frac{l_c}{2} \cdot sin(\alpha) = \frac{87.435}{2} \cdot sin(30^o) = 21.8546\ mm$$

Next, we need the distances from the datum line to the components:

$$Y_{shell} = 0\ mm, \quad Y_{cone} = 21.8543\ mm, \quad Y_{ring} = -56\ mm$$

Now we can compute the first moment of area of the components about the datum line:

$$M_{shell} = \ A_{shell} \cdot Y_{shell} = 976.4436 \cdot 0 \qquad = \qquad 0\ mm^3$$

$$M_{cone} = \ A_{cone} \cdot Y_{cone} = 1049.222 \cdot 21.8543 = \quad 22\ 930\ mm^3$$

$$M_{ring} = A_{ring} \cdot Y_{ring} = 1200 \cdot -56 \qquad = -67\,200\,mm^3$$

We need the moment of inertia of the components about the datum line. We can only compute them for the non-slanting (cone) component. We deal with the cone in a moment:

$$I_{oshel} = M_{shell} \cdot Y_{shel} + \frac{l_s \cdot t_s{}^3}{12} = 0 \cdot 0 + \frac{81.3703 \cdot 12^3}{12} \qquad = 11\,717.32\,mm^4$$

$$I_{oring} = M_{ring} \cdot Y_{ring} + \frac{t_{ring} \cdot h^3}{12} = -67\,200 \cdot -56 + \frac{12 \cdot 100^3}{12} \qquad = 4\,763\,200\,mm^4$$

Now we must compute the moment of inertia of the cone. The method is in the Appendix.

First, we compute the moment of inertia about its centroid (see Appendix):

$$I_{cone\_g} = \frac{A_{cone}}{12}\left(l_c{}^2 \cdot sin(\alpha)^2 + t_c{}^2 \cdot cos(\alpha)^2\right) \text{ please see the Appendix}$$

$$= \frac{1200}{12}(87.4352^2 \cdot sin(30)^2 + 12^2 \cdot cos(30^o)^2)$$

$$= 176\,488\,mm^4$$

We also need the moment of inertia of the cone about the datum:

$$I_{cone\_y} = A_{cone} \cdot y^2 = 1049.222 \cdot 21.8546^2 \qquad = 501\,131\,mm^4$$

Now add them together to get the total moment of inertial about the datum:

$$I_{ocone} = I_{cone\_g} + I_{cone\_y} = 176\,488 + 501\,131 = 677\,619\,mm^4$$

Now total the moments of inertia, the areas and first moments of inertia about the datum:

$$I_o = 11717.323 + 677619 + 4763200 \qquad = 5\,452\,536\ mm^4$$

$$A_o = 976.4436 + 1049.222 + 1200 \qquad = 3\,225.666\,mm^2$$

$$M_o = 0 + 22930 - 67200 \qquad = -44\,269.7\,mm^3$$

We need the position of the centroid of the combined components about the datum:

$$y_o = \frac{M_o}{A_o} = \frac{-44\,269}{3\,225.66} = 13.7242\,mm$$

At last, we can compute the moment of inertia of the cone-to-shell junction:

$$I_{actual} = I_o - A_o \cdot y_o{}^2 = 5\,452\,536 - 3\,225.666 \cdot 13.7242^2$$

$$= 4\,844\,970\,mm^2$$

$$I_s{}' = 710\,395.5\,mm^4 \text{ required – see above}$$

The moment of inertial is more than adequate.

In PV Elite, there appears a table to find the section properties. Here is the table as set out in Excel, which is the easiest way of doing section properties analyses:

Element	B	D	y	A=B.D	M=A.Y	Io=M.y+B.D^3/12
Shell	81.37	12	0	976.44	0.00	11717.28
Cone	87.4352	12	21.8546	1049.22	22930.34	677619.00
Ring	12	100	-56	1200.00	-67200.00	4763200.00
	y = M/A =	-13.7242	mm	3225.66	-44269.66	5452536.28
	I = Io - A*y^2=	4844970	mm^4			

Analysis is the small end of the cone:

Again, looking at Figure 3, we need the effective length of the shell to the right of the small end of the cone.

$$L_S = 2000 + 50 + 83.333 = 2133.333\ mm$$

These are the required thickness for external pressure for the cone and the small diameter shell (as a reminder):

$t\ \ = 4.381\ mm$ for the small diameter shell

$t_r = 3.480\ mm$ for the cone

According to UG-99 $t_r = \frac{t_r}{\cos(\alpha)} = 4.0184\ mm$ as we derived earlier above

Compute the decay lengths of the shell and the cone:

$$l_s = 0.55 \cdot \sqrt{D_S \cdot t_s} = 0.55 \cdot \sqrt{1020 \cdot 10} = 55.5473\ mm$$

$$l_c = 0.55 \cdot \sqrt{D_s \cdot \frac{t_c}{cos(\alpha)}} = 0.55 \cdot \sqrt{1020 \cdot \frac{12}{cos(30^o)}} = 65.3844\ mm$$

Compute the required area of the junction:

$$A_{es} = 0.55\left(\sqrt{D_S \cdot t_s} \cdot (t_s - t) + \sqrt{D_s \cdot \frac{t_c}{cos(\alpha)}} \cdot (t_c - t_r)\right)$$

$$= 0.55\left(\sqrt{1020 \cdot t_s} \cdot (10 - 4.381) + \sqrt{1020 \cdot \frac{12}{cos(30^o)}} \cdot (12 - 4.0184)\right)$$

$$= 788.54\ mm^2$$

Actual area available in the junction:

$$A_{avail} = l_s \cdot t_s + l_c \cdot t_c = 55.5473 \cdot 10 + 65.3844 \cdot 12$$

$$= 1028\ mm^2$$

The area is adequate

We must now compute the required moment of inertia of the junction:

$$Q_S = \frac{P_e \cdot R_S}{2} = \frac{0.1 \cdot 510}{2} = 25.5\ {}^{N}/_{mm}$$

$$A_{rs} = \frac{k \cdot Q_S \cdot R_S \cdot tan(\alpha)}{S_s \cdot E_1} = \frac{1.0 \cdot Q_S \cdot 25.5 \cdot tan(30^o)}{138 \cdot 1.0}$$

$$= 54.395\ mm^2$$

The area of the junction is adequate

This section consider the require moment of inertia:

$$A_{TS} = \frac{L_S \cdot t_S}{2} + \frac{L_S \cdot t_c}{2} = \frac{2133.33 \cdot 10}{2} + \frac{2\ 133.33 \cdot 12}{2} \quad = 23\ 466\ mm^2$$

$$N = \frac{-R_S \cdot tan(\alpha)}{2} + \frac{L_S}{2} + \frac{{R_L}^2 - {R_S}^2}{6 \cdot R_S \cdot tan(\alpha)}$$

$$= \frac{-510 \cdot tan(30^o)}{2} + \frac{2\ 133.33}{2} + \frac{912^2 - 510^2}{6 \cdot 510 \cdot tan(30^o)} = 1\ 243.132\ mm$$

$$F_S = P_e \cdot N = 0.1 \cdot 1\ 243.131 \qquad = 124.313\ {}^{N}/_{mm}$$

$$B = \frac{3}{4} \cdot \frac{F_S \cdot D_S}{A_{TS}} = \frac{3}{4} \cdot \frac{124.313 \cdot 1020}{23\ 437} \qquad = 4.0525\ MPa$$

At this point, we need the value of A from Figure CS-2 from ASME Section II – D. However, the value of B falls off the chart, so must it be computed:

$$A = \frac{2 \cdot B}{E_s} = \frac{2 \cdot 4.0525}{198\ 163} = 0.000\ 0409\ {}^{mm}/_{mm} \quad \text{a very small value!}$$

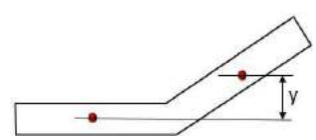
Now we can compute the required moment of inertia of the junction:

$$I_S{}' = \frac{A \cdot D_S \cdot A_{TS}}{10.9} = \frac{0.000\ 0409 \cdot 1020 \cdot 23\ 467}{10.9}$$

$$= 91\ 614\ mm^4$$

Again, referring to the Appendix, the actual moment of inertia of the junction is needed:

$$y = \frac{l_c}{2} \cdot sin(\alpha) = \frac{65.3844}{2} \cdot sin(30^o) = 16.3429$$



$$A_{cone} = l_c \cdot t_c = 65.3844 \cdot 12 = 784.6126\ mm^2$$

The moment of inertia of the cone about its own centroid:

$$I_{cone-g} = \frac{A_{cone}}{12} \cdot \left({l_c}^2 \cdot sin(\alpha)^2 + {t_c}^2 \cdot cos(\alpha)^2\right) \quad \text{about it's centroid per the Appendix}$$

$$= \frac{784.6126}{12} \cdot (65.3844^2 \cdot sin(30^o)^2 + 12^2 \cdot cos(30^o)^2)$$

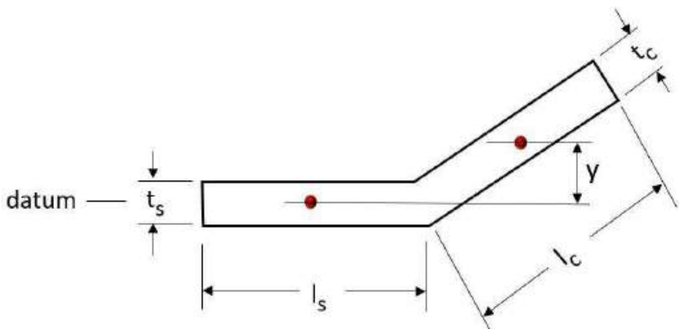
$$= 76\ 917\ mm^4$$

The moment of inertia of the cone about the datum line:

$$I_{cone-y} = A_{cone} \cdot y^2 = 784.6126 \cdot 16.3429^2 = 209\ 562\ mm^4 \text{ about the datum}$$

We need the Area, First Moment of Area and the Inertia about the datum for the two components:

Referring to this illustration below, find the Area, first moment of area and the moment of inertia of the components of the junction region:



$$A_{shell} = l_s \cdot t_S = 55.5473 \cdot 10 = 555.4728\ mm^2$$

$$A_{cone} = l_c \cdot t_c = 65.3844 \cdot 12 = 784.6126\ mm^2$$

$$M_{shell} = A_{shell} \cdot y_{she} \quad = 555.4728 \cdot 0 \quad = 0\ mm^3$$

$$M_{cone} = A_{cone} \cdot y = 784.6126 \cdot 16.3429 = 12\,822.857\ mm^3$$

$$Io_{shell} = M_{shell} \cdot y_{shell} + \frac{l_s \cdot {t_s}^3}{12} = 0 \cdot 0 + \frac{55.5473 \cdot 10^3}{12} = 4\,628.9\ mm^4$$

$$Io_{cone} = M_{cone} \cdot y_{cone} + I_{cone-} \quad = 12\,822.857 \cdot 16.3429 + 76\,917$$

$$= 286\,479\ mm^4$$

Now we can add the areas, first moment and the moments of area:

$$A = 555.4728 + 784.6126 \ = 1\,340.08\ mm^2$$

$$M = 0 + 12\,822.857 \qquad = 12\,822.857\ mm^3$$

$$Io = 4358.9 + 286479 \qquad = 291\,108\ mm^2$$

We can now obtain the centroid for the shell and cone combined, and obtain the final moment of inertia for the cone-to-shell junction:

$$y = \frac{M}{A} = \frac{12\,822.857}{1\,340.08} = 9.5687\ mm$$

$$I_s \ = Io - A \cdot y^2 = 1340.08 \cdot 9.5687^2 = 168410\ mm^4$$

$$I_s{}' = 91\,614\ mm^2$$

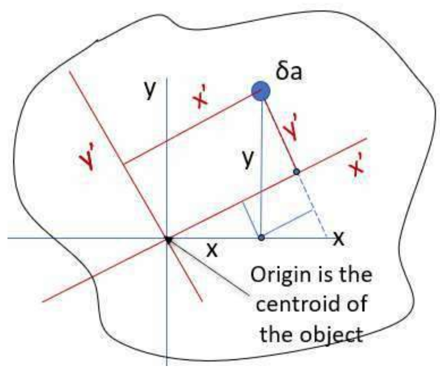
The moment of inertia is more than adequate. As before, we can set this out Excel, which all Excel to crunch all the number:

Element	B	D	y	A=B.D	M=A.Y	Io=M.y+B.D^3/12
Shell	55.5473	10	0	555.473	0	4628.941667
Cone	65.3844	12	16.3429	784.6128	12822.849	286480
	y = M/A =	9.568677	mm	1340.086	12822.849	291108.9417
	I = Io - A*y^2=	168411.2	mm^4			

APPENDIX

This is the basic theory behind obtaining the section properties of a rectangular element the is at a angle to the horizontal. Another way of stating this procedure is – transform of axes. We have two sets of orthogonal axes subtending and angle α at the origin. The origin is the centroid of object, or which a small element δa is considered. The reader might recall that the method is theme on the parallel axis theorem.

We are considering δa from the perspective of the X'-Y' axis, when the position of the element is known from the X-Y axis. We are going to perform the transform:



$$x' = \quad x \cdot cos(\alpha) + y \cdot sin(\alpha)$$

$$y' = -x \cdot sin(\alpha) + y \cdot cos(\alpha)$$

However, we need the moment IXX about the X-X horizontal axis, therefore we need the above two formulae expressed in terms of x and y, converted from x' and y':

Multiply the first equation by sine and the second by cosine:

$$x' \cdot sin(\alpha) = \quad x \cdot cos(\alpha) \cdot sin(\alpha) + y \cdot sin(\alpha)^2$$

$$y' \cdot cos(\alpha) = -x \cdot sin(\alpha) \cdot cos(\alpha) + y \cdot cos(\alpha)^2$$

Add the two equations:

$$y \cdot (\sin(\alpha)^2 + cos(\alpha)^2) = x' \cdot \sin(\alpha) + y' \cdot cos(\alpha)$$

Which reduces to:

$$y = x' \cdot \sin(\alpha) + y' \cdot cos(\alpha)$$

The expression becomes to:

$$I_{XX} = \frac{A}{12} \cdot (x^2 \cdot sin(\alpha)^2 + 2 \cdot x \cdot y \cdot \sin(\alpha) \cdot cos(\alpha) + y^2 \cdot cos(\alpha)^2)$$

but $\quad A \cdot x \text{ is the first moment about the centroid which must be zero, by definition}$

the express thus reduced to: $\quad I_{XX} = \frac{A}{12} \cdot (x^2 \cdot sin(\alpha)^2 + y^2 \cdot cos(\alpha)^2)$

The number ‘12’ in the denominator only applies to a rectangular block. If it were a circular or triangular section, the denominator would be a different number.

We now have an expression for determining the moment of inertia of a sloping component about it centre of area or the centroid.

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